PHYSICS OF FINANCIAL MARKETS: A VIEW FROM BARCELONA

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The Physics and Finance group in University of Barcelona exists since 1997. Prof Jaume Masoliver is the leader and our work has been mainly devoted to the modelling of financial data time series using stochastic differential equations or using the so-called Continuous Time Random Walk approach. Since then we have published more than 40 papers on financial markets from a Physics department with many collaborations with researchers from not only Physics departments but also from Economics departments. It is herein explained how we start working as a group on this field and which are our main contributions and activities. To conclude, a brief outline of other Spanish contributions and activities is also presented.

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Our story begins, I would say, as many others experiences in Econophysics from casual and very particular facts often with family or friends in between. I still remember how I decided to start my PhD in Physics and how I felt attracted to this heterodox and multidisciplinar research field. This was in 1996. The year I finished my graduate studies in Physics. My father who has been working for almost 40 years as a fund manager and trader explained me in a rather unconcrete manner that a large amount of physicists were hired in Dow Jones as something called quants[1]. He told me that they were very well considered mainly for two reasons: they know more statistics and mathematics than a regular economist and they provide quick answers without the need of building compact theories and robust theorems as mathematicians typically do.

Few months later, the Nobel prize 1997 was awarded to Robert C. Merton and Myron S. Scholes "for a new method to determine the value of derivatives". The media trying to explain the very curious story behind their finding explained that the partial differential equation lying at the heart of Black-Scholes(-Merton) theory was precisely a widely used equation in Physics: the backward Fokker Planck equation[2,3]. This second link between Finance and Physics definitively motivated me to start my PhD in what is called Econophysics. This field appeared to me very attractive since it has a perfect balance between theory and empirics. Observations are truly near (on your desk and in your pc) and can be easily contrasted thanks to the birth of electronic markets in early 1990s.

The Stochastic Dynamics and Transport Phenomena group and his leader Prof Jaume Masoliver accepted me at that moment to start a PhD research applying stochastic processes and random walks to the study of financial markets. The group did publish several relevant works on Gaussian colored noise in the context of resonant phenomena[4] or the challenging but classic first-passage problem[5,6] among others with practical interest in reaction dynamics in chemistry, light transport in optics or even tumor growths in biomedicine. Our purpose was to shift this powerful methodologies and know-how to the study of such a particular human activity like speculative markets. We were specially looking at the financial time series and to the possibilities of modelling this paths using stochastic dynamics. Our approach was to provide a different insight

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to financial mathematicians and econometricians remaining closer to empirics and we were aimed to follow the path opened with several pioneering works in Econophysics. For many reasons, we were particularly interested by the pioneering works of Eugene H. Stanley[7], Rosario N. Mantegna[8], and Jean-Philippe Bouchaud[9].

From that moment, the whole group on Physics and Finance: Jaume Masoliver, Josep Maria Porrà, Miquel Montero and myself started to work in Econophysics in different ways. The first temptative was to run a research on bonds for a trading company. The following contributions also included research in academia. My colleagues in the group published a first paper providing a model to the stock price statistics observations described by Mantegna and Stanley[7,10]. The model is an extension of the diffusion process adding a shot-noise corresponding to the abnormal vibration in price that Merton proposed in 1976[11]. The new contribution describes self-similarity and fat tails distribution of prices observed in empirical data. One year later, we presented a description of the Black Scholes pricing method[2] with the differential calculus aimed by physicists: the Stratonovich convention[12]. The contribution is nothing more (but not for this less important) than a reinterpretation of the Black-Scholes option price derivation but with a friendly language from physicist point of view. Subsequent contributions were mostly focused on option pricing and how to add easily the effects of time correlation and fat tails in the Black-Scholes option pricing methodology[13,14,15].

Other contributions were following some well-known techniques coming from Mathematical Finance like Malliavin Calculcus of American Option computational methods. These were made with the collaboration of Miquel Montero and myself with Prof Arturo Kohatsu who at that time was based in the Departament of Economics in Universitat Pompeu Fabra[16–19].

The current paper is however focussed mainly on our more important research areas: the stochastic volatility modelling and the continuous time random walk approach. Other contributions not fully motivated in the following chapters are those concerning time series analysis with Diffusion Entropy (with Luigi Palatella and Ingve Simonsen from Dresden University)[21,22,23], agent based modelling for the order book and price dynamics[24] with Prof Giulia Iori from City University and Prof Carl Chiarella from Sidney University and downside risk analysis for hedge funds[20]. We also have studied the effect of taking the difference of stock prices instead of taking the difference of logarithm of prices[25]. At that time some physicists were still taking differences instead of logarithmic differences. The choice may lead to important errors in the statistical analysis[25].

Stochastic Volatility Models

The multiplicative diffusion process known as the geometric Brownian motion has been widely accepted as a universal model for speculative markets. The model, suggested by Bachelier[26] in 1900 as an ordinary random walk and redefined in its final version by Osborne in 1959[27], presupposes a constant volatility σ, that is to say, a constant diffusion coefficient D. However, and especially after the 1987 crash, there seems to be empirical evidence, embodied in the so-called stylized facts[28], that the assumption of constant volatility does not properly account for important features of markets. It is not a deterministic function of time either as might be inferred by the evidence of nonstationarity in financial time series but a random variable. In its more general form one therefore assumes that the volatility is a given function of a random process.

The 2002 is an important year for us since we opened a new research line. Thanks to the observation by Bouchaud and his colleagues of the leverage effect (or also called return-volatility asymmetric correlation)[29]. The phenomena was already documented in the literature but this was the first time we saw it in a quantitative way as physicists like to observe temporal correlations. We observed that the Stochastic Volatility (SV) models, which assumes a random volatility with its own diffusion process, are able to explain in a very natural way the leverage effect and other statistical observations related to volatility. These sort of models were proposed by several financial mathematicians specially after the 1987 financial derivatives crash in order to include fat tails behaviour in the option price evaluation. However, they were not deeply explored as models able to reproduce the financial time series. Research was mainly devoted to obtain a better estimation of option prices.

We noticed that the SV models (also called in Physics random diffusion[30]) is a very natural choice for explaining the leverage effect. Even the simplest, the Stein and Stein model[31] assuming that volatility follows an Orstein-Uhlenbeck process, gives reason to this very peculiar return-volatility correlation. We investigated and confronted their abilities with daily data from Dow Jones[32,33] and other financial markets[34]. We were also looking for other stylized facts we finally got the conclusion that among the simplest models the exponential Ornstein-Uhlenbeck model[35] is the best. This model is able to explain simultaneously the leverage correlation with a characteristic time scale of few weeks and the volatility autocorrelation which is typically important for time lags longer than a year[36]. See an alternative model in Ref.[37] with Bouchaud as a coauthor.

After these findings, we drew our attention to new tools of quantifying risk. Extreme-value problems have a clear financial interest apart from the obvious relation to the classic ruin problem. As an example, among others, let us mention the so-called leverage certificates (LCs) which are structured products offering a nonzero pay-off only if the underlying asset does not escape from a pre-established domain over a certain time window. Although usually sold as products insensitive to volatility, LCs are very sensitive to skewness and kurtosis. On the other hand, SV models result in fat tailed distributions for the return and show clustering in the volatility, two well-established facts in empirical data which are closely related to skewness and kurtosis. For this reason, the solution to the escape problem under stochastic volatility can be used to derive a more precise price than that of the Wiener process for a wide class of LC products.

The study of first-passage and exit problems have a long and standing tradition in physics, engineering, and natural sciences. Perhaps the most important example of an exit problem in physics is provided by the Kramers problem where one studies the possible escape, owing to noise, of a system from a stable estate. Classical examples of first-passage problems are the collapse of mechanical structures because of random external vibrations which attain an extreme amplitude beyond the stability threshold, or the false alarm problem where internal fluctuations induce the current or voltage of an electric circuit to reach a critical value for which an alarm is triggered.

We first addressed a partial aspect of the problem: that of extreme times for the volatility regardless the value of the price return[38]. We afterwards focussed on a specific model, the so-called Heston model[39], which for mesoscopic time windows (longer than a day but shorter than a year) provide a realistic picture of the stock price statistics. We first solved the overall escape problem associated with both return and volatility. We obtained not only the exact expression of the mean escape time (MET) but also the exact survival probability[40]. We solved the first-passage problem for the Heston random diffusion model as well[41]. We obtained exact analytical expressions for the survival and the hitting probabilities to a given level of return. We obtained approximate forms of these probabilities which prove, among other interesting properties, the nonexistence of a mean-first-passage time. One significant result is the evidence of extreme deviations –which implies a high risk of default– when certain dimensionless parameter, related to the strength of the volatility fluctuations, increases. We confronted the model with empirical daily data and we observe that it is able to capture a very broad domain of the hitting probability. We believe that this may provide an effective tool for risk control which can be readily applicable to real markets both for portfolio management and trading strategies. This is still an ongoing task. We are now particularly focussing our energy on intraday and high frequency data.

Other contributions within the SV modelling concern visualizing with maximum likelihood methods (in collaboration with Zoltan Eisler now working in CFM research area with Bouchaud) the hidden path of the random volatility[42], or giving price to option contracts through simulation[43] or through Black-Scholes theory[44–47] assuming different SV underlying dynamics. One of these contributions has been done with Prof Ronnie Sircar from Princeton University.

Continuous Time Random Walk Approach

The continuous time random walk (CTRW), first introduced by Montroll and Weiss[48], has become a widely used tool for studying the microstructure of random process appearing in a large variety of physical phenomena. These range from transport in disordered media, earthquake modelling and even solar surface kinetics, to name just a few. Our aim has been to extend the fields of application of the CTRW analysis by including the dynamics of financial markets. In fact, this formalism was the first tentative model known in finance, having been suggested by Bachelier[26] to describe stock market dynamics and give a price for a European call option. In fact, Bachelier modelled the price evolution assuming that prices change one unit at each time step with a probability p of going up and $1 - p$ of going down. Thus, there are only two possible events. This process is called the binomial model and is the simplest random walk.

Despite this promising fact, the CTRW had been hardly known among financial analysts for decades. Physicists have recently provided only a few examples of CTRWs applied to finance. Thus, the papers by Scalas *et al.* in 2000[49] were among the first works addressed to this issue. We have also contributed in further developments since the group has a long experience in the CTRW with the collaboration of Prof George Weiss[50–52]. Perhaps one of the most solid reasons in favor of CTRW models is that they provide general expressions for the distribution of prices at time t in terms of two auxiliary densities that can be estimated from data: the probability density function (pdf) of the pausing time between ticks $\psi(t)$ and the density for the magnitude of the price increment at a given tick $h(x)$.

Other quantities, such as the distribution of daily or longer-time prices based on two probability density functions can be obtained using the CTRW formalism[50].

This in turn allows for the possibility of dealing with inverse problems, that is, estimating from the observed daily or longer-time data the forms of the microscopic functions ψ(t) and $h(x)$[51,52]. This is useful since in many practical situations, one only has, at most, daily data. In this case our formalism enables us to determine features of the otherwise unknown microscopic structure of the financial process.

We studied theoretical and empirical aspects of the mean exit time (MET) of financial time series[53–55] within the framework of continuous time random walk. Reference[55] is done with the group of Prof Mantegna from Palermo. We empirically verify that the mean exit time follows a quadratic scaling law and it has associated a prefactor which is specific to the analyzed stock. We perform a series of statistical tests to determine which kind of correlation are responsible for this specificity. The main contribution is associated with the autocorrelation property of stock returns. Following these approach we dealt also with correlations and cross-correlations in this context but also looking at the distribution of prices[56,57].

We also studied the statistics of the waiting time between transactions in many ways[50,58,59]. In this sense, we could stress an approach directly inspired by the Valley Model[60] which was initially proposed to describe the powerlaw relaxation of photocurrents created in amorphous materials with the group of Prof Ryszard Kutner from University of Warsaw. It is well known that the inter-event statistics observed in these contexts differs from the Poissonian profile by being a long-tailed distributed with resting and active periods interwoven. Understanding mechanisms generating consistent statistics has therefore become a central issue. The approach represents an analytical alternative to models of nontrivial priority that have been recently proposed. Our analysis also goes one step further by looking at the multifractal structure of the interevent times of human decisions. We observe that empirical data describe a subtle multifractal behavior[58]. Following the same approach we analyzed the thermodynamical consequences for these observations[61].

Other studies are based on the link of option pricing and CTRW framework[62].

Econophysics in Spain: Some Concluding Remarks

After 13 years of experience in this field we have published more than 40 papers in peer-review Physics journals and Economics journals. Since we started two of us have obtained a permament position in Departament de Física Fonamental of Universitat de Barcelona inside Condensed Matter area of knowledge. There are some other experiences like ours in Spain although they are more shifted to social dynamics or to complex networks more likely to use simulations and numerical methods. There is no other group, as far as I know, fully devoted on the study of financial markets with a physicist perspective with such a large number of permanent staff involved. Some relevant groups from Barcelona, Palma de Mallorca (the quite big Institute for Cross-Disciplinary Physics and Complex Systems) or from Madrid (Universidad Carlos III although from a Department of Mathematics) have published respectively interesting papers on the world trade web flows[63], on transmission of information and herd behavior in financial markets[64] and on cooperation if this can be understood as economics as well[65]. It is worth mentioning the work by Esteban Moro has contributed in the study of minority games[66], decision making in financial trading mostly based on information arised from the order book[67-69].

In a larger scale, one can add that some institutions are teaching Econophysics inside Multidisciplinar Physics Master Programms like the one we teach in "Computational and Applied Physics"' (Universitat Politècnica de Catalunya and Universitat de Barcelona) or the "Physics of Complex Systems"' (Universidad Complutense, Universidad Nacional de Educación a Distancia, Universidad Politécnica de Madrid and Universidad Carlos III Madrid). However, there is no post-graduate course program fully devoted to this area. It is finally relevant to say that it already exists a network of about 30 senior researchers on physics of economic and social systems (econosociophysics) from several universities which enable us to be in contact with other Spanish groups and share experiences in this young multidisciplinary area.

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